

Geometries with integrable singularity – black/white holes and astrogenic universes

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Abstract

We briefly review the problem of generating cosmological flows of matter in GR (the genesis of universes), analyze models' shortcomings and their basic assumptions yet to be justified in physical cosmology. We propose a paradigm of cosmogenesis based on the class of spherically symmetric solutions with *integrable* singularity $r = 0$. They allow for geodesically complete geometries of black/white holes, which may comprise space-time regions with properties of cosmological flows.

1 Introduction

I.D. Novikov's paper [1] where he developed the model of collapsing electrically charged sphere that gives rise to expansion in another universe pioneered scientific research of the cosmogenesis problem, that is, the problem of generating expanding matter flows in General Relativity (GR). The studies took various trends. Widely discussed became bouncing models and the birth of the universe from “nothing” with matter entering a highly dense state with de Sitter symmetry and then leaving this state by tunnelling into the stage of cosmological expansion (see, for example, [2]-[8]). The models with continuous phase transitions turning collapse into expansion required matter with an exotic equation-of-state, including the matter violating the weak energy condition [9]. These and some other models of cosmogenesis studied so far (see, for example, [10]) either started from the initial highly dense state with symmetry close to that of the de Sitter space-time or assumed that the similar state would emerge as a result of non-linear quantum-gravitational effects, yet unknown to the modern science, in the matter of collapsing object.

In this paper we propose another approach to the problem of generating cosmological flows based on the class of general spherically symmetric GR solutions with 2+2 split and *integrable* singularity $r = 0$. The latter allows us to continue the radial geodesics

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with respect to the proper affine parameter, thus, constructing geodesically complete geometries of black/white holes. These solutions require the presence of effective matter in T -regions *outside* the body of collapsing object and make it clear, thanks to the continuous geodesics, that after the inversion moment $r = 0$ there forms an expanding white-hole T -region with all the properties of cosmological flow. We show that the eternal black/white hole metrics is sourced by an effective matter with negative longitudinal pressure $p < 0$ located in the vicinity of the spatial hypersurfaces $r = 0$. We present a toy model of the geometry invariant with respect to the sign of r and make a suggestion that the effective matter with similar properties also fills T -regions of astrophysical black holes. This leads us to the following hypothesis of cosmogenesis: an object collapsing into black hole generates ultra-high curvature outside the collapsing body, which, in its turn, induces violent particle creation, thus, transferring the initial momentum to the newly created matter. The latter having passed to the T -region of the white hole gives rise to an expanding cosmological flow. If driven by inflation, the flow can grow to become quasi-Hubble in a large volume. Such *astrogenic universes* may emerge inside black/white holes which are naturally generated on the final stage of evolution of stars, clusters and other compact astrophysical objects in the maternal universe.

The next section deals with basic postulates and assumptions of the cosmogenesis models. In Sects. 3 and 4 we study properties of spherical solutions with respect to inversion of the spherical reference frame and define classes of geodesically complete geometries. Sects. 5 and 6 deal with the problem of the black/white-hole source: we argue that the effective matter is inevitably present in T -regions of the holes and develop toy models of integrable singularities by introducing triggered phase transitions. In the last section we summarize and discuss the results.

2 A brief review of the problem

Since GR came into being, the scientific community has had various opportunities to verify that it gives a viable description of phenomena that include strong gravitational fields and relativistic velocities. Its experimental basis once consisting of the three classical GR effects (perihelion precession, deflection of light and red shift) has recently acquired one of its crucial contributions – the Cosmological Standard Model (CSM) of the visible Universe. Although some theoretical premises of the model (e.g. hypotheses of dark matter and dark energy) may inspire certain GR modifications, in the conditions encountered in observational cosmology Einstein’s theory of gravity provides a high degree of precision as soon as the CSM is challenged by experimental data [11].

The cornerstone of the standard model is the cosmological principle synonymous to the large-scale Friedmannian symmetry. Extrapolating the CSM to the past respects this symmetry and leads to the geometry of the early Universe in the form of a rapidly expanding quasi-Hubble matter flow that comes out of the state with ultra-high space-

time curvature and density [12]. If we try to extrapolate this state within the general-relativistic framework even closer to the ultimate zeroth instant of time, we run into the singular point where the metric space itself does not exist. The same happens in the black-hole physics where, however, the singularity is, vice versa, the final stage of a collapsing object.

As a rule, if a theory exhibits singularities they are a mere consequence of neglecting some physical phenomena, or, in other words, a consequence of idealization. Therefore, the singular states in GR solutions indicate the limits of the modern theory of gravity. The singularity problem arising in the GR (both in the black-hole physics and cosmology) is caused by our inability to fully understand what is gravitation and how it interacts with matter. The theorems known from the 60's imply that world lines of test particles cannot go beyond the singularity because of infinite tidal forces [13]. The theorems are based on some matter properties that are required *a priori*. These are the dominant, strong and weak energy conditions. The first two of them may not be fulfilled in quantum-gravity physical processes where gravitation and matter intensely interact. Besides, one should differ the problem of existence of continuous metric space-time itself (where a photon or test particle propagate) and the problem of divergence of some curvature or energy-momentum components. The current analysis demonstrates that there are neither observational nor theoretical grounds proving singular states inevitable.

Inflationary models [14] explained the large size and the Friedmann symmetry of the observed matter flow, thus, providing a theoretical basis for the cosmological principle itself. The problem of initial conditions, however, continued unsolved. As a matter of fact, one postulate (of isotropy and homogeneity) was substituted by another. In other words, *where do the ultra-high density and initial impetus launching the expansion come from?* For example, in different models of inflation [10] new physical fields (often unknown to the modern particle physics) are introduced in the ultra-dense state from the very beginning. The birth of the Universe from "nothing" also involves the notion of highly dense "false" vacuum. In so-called bouncing models having been developed for more than 40 years now the problem of initial conditions is replaced for specific matter properties at high densities, and again the Friedmann symmetry is assumed.

In this paper we make use of the fundamental scientific principle that states that any physical solution describing nature must contain only such observable quantities that remain finite. It leads us to consider realistic models of black/white holes with *smoothed* metric singularities (the metric potentials remain finite). This allows us to constrain the tidal forces (despite a possible divergence of some curvature components) and construct a geodesically complete metric space-time in dynamical models incorporating solely the general physical principles – energy-momentum conservation, a wide choice of the equation-of-state, the weak energy condition. Thus, now the geodesics pass to the *T*-region of a white hole rather than end in the singularity. From this point we arrive to the hypothesis that any black hole originated from the collapse of an astrophysical object may

give birth to a new (daughter or astrogenic) universe. The conjecture naturally solves all of the three above-mentioned problems of cosmogenesis:

- The ultra-high curvature and density on the initial stage of cosmological evolution are achieved as a consequence of superstrong and highly variable gravitational fields that exist inside the black/white hole and generate the proper matter of the daughter universe.
- The initial push to the expansion of the generated matter (the Big Bang) is given by the T -region of the white hole. The initial cosmological impetus is, hence, of pure gravitational nature and is one of the manifestations of gravitational (tidal) instability.
- The T -region symmetry of the black hole outside the maternal body of the collapsing object is that of an anisotropic cosmology. It is transferred to the white-hole T -region and can be made isotropic by known inflationary mechanisms.

None of unknown physical effects which are in play at high densities can prevent us from using dynamical equations in the form of the Einstein equations $G^\mu_\nu = 8\pi GT^\mu_\nu$, where any high-energy and high-curvature geometrical modification is included in the right-hand side and ascribed to the *effective* energy–momentum tensor T^μ_ν , thus, containing both material and, in part, spacetime degrees of freedom (see, for example, [16]). We do not calculate one-loop and other "corrections" that are of no use in this case, since the processes in question involve *non-linear* phase transitions taking place in the effective matter at high curvature. We simulate this non-linearity phenomenologically using dynamical solutions with triggered matter generation and do not constrain the equation-of-state. The basic geometrical object is the averaged metric tensor $g_{\mu\nu}$, which is used to construct the left-hand side of the equations according to the GR rules. The equations obtained hold in the entire space–time, and the manifold does not contain punctured points.

3 Properties of geometries with respect to the inversion $r \rightarrow -r$

In the Schwarzschild metrics (M is the external mass)

$$ds^2 = \left(1 - \frac{2GM}{r}\right) dt^2 - \frac{dr^2}{1 - \frac{2GM}{r}} - r^2 d\Omega, \quad (1)$$

the quantity $r > 0$ [17] plays two roles. On one hand, it is the curvature radius of a 2-dimensional sphere with the squared line element $r^2 d\Omega$. On the other hand, it is one of the

coordinates. In order to discriminate one from the other, let us present the metrics of an arbitrary spherically symmetric 4D space–time split into a pair of 2D spaces [18, 19, 20]:

$$dX^2 = n_{IJ} dx^I dx^J \quad (2)$$

and

$$dY^2 = \gamma_{ij} dy^i dy^j \equiv r^2 d\Omega, \quad (3)$$

where the functions n_{IJ} and r depend on the variables $x^I = (x^1, x^2) \in \mathbb{R}^2$ and are *independent* of the internal 2D coordinates y^i of the closed homogeneous and isotropic 2-surface \mathbb{S}^2 of the unit curvature $d\Omega = \omega_{ij} dy^i dy^j$, $\gamma_{ij} \equiv r^2 \omega_{ij}$. If the coordinates are chosen to be angular, $y^i = (\theta, \varphi)$, we have $\omega_{ij} = \text{diag}(1, \sin^2 \theta)$ with $\theta \in [0, \pi]$ and $\varphi \in [0, 2\pi)$.

By choosing the four coordinates of the covering grid one can turn four non-diagonal components of the full metric tensor into zero $g_{Ii} = 0$ and write it in the orthogonal reference frame $x^\mu = (x^I, y^i)$:

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = dX^2 - dY^2, \quad (4)$$

where $g_{\mu\nu} = \text{diag}(n_{IJ}, -\gamma_{ij})$ is the metrics of the spherically symmetric geometry in the orthogonal split $2 + 2$. The energy–momentum tensor corresponding to (4) is $T_{\mu\nu} = \text{diag}(T_{IJ}, -p_\perp \gamma_{ij})$, where p_\perp is the transversal pressure.

At this point the metric potential r in (3) can be introduced in the invariant manner as a radius of the *internal* curvature ρ of the closed Y -space, where

$$R_{ij}^{(Y)} = \rho \gamma_{ij} \quad \text{and} \quad \rho \equiv \frac{1}{2} R^{(Y)} = r^{-2} \quad (5)$$

are the Ricci tensor and scalar constructed from the metrics γ_{ij} . By definition, the 2-space γ_{ij} and its internal curvature ρ are invariant with respect to interchanging r and $-r$ while the *external* curvature of Y depends on the sign of r and determines the orientation and evolution of the surface in the space–time (4):

$$\mathcal{K}_{ijI} \equiv \frac{1}{2} \gamma_{ij,I} = \mathcal{K}_I \gamma_{ij}, \quad \mathcal{K}_I \equiv \frac{1}{2} \gamma^{ij} \mathcal{K}_{ijI} = \frac{r_{,I}}{r}, \quad (6)$$

where the comma in the subscript stands for the partial derivative with respect to x^I . This fact becomes obvious in the coordinates where one of the variables x^I is identically equal to r .

Changing the sign $r \rightarrow -r$ with θ and φ staying unchanged is equivalent to inverting¹ the spherical reference frame (4). In the regions with negative r with no matter ($T_{\mu\nu} = 0$) r in the solution (1) is to be substituted with the absolute value $|r|$. The parameter M continues to stand for the external (measured in the asymptotically flat space) hole mass. The inversion results in the black hole transforming into the white hole (and vice versa).

¹One can fulfil the operation holding the condition $r \geq 0$, but sacrificing the continuity of the angular coordinates at $r = 0$: $\theta \rightarrow \pi - \theta$, $\phi \rightarrow \phi + \pi$. Note that in the regions where r does not change its sign one can always restore the condition $r > 0$ by renaming the coordinates θ and φ .

In the general case when the matter is present, the metrics (4) can be written in the Euler orthogonal coordinates:

$$ds^2 = N^2(1 + 2\Phi) dt^2 - \frac{dr^2}{1 + 2\Phi} - r^2 d\Omega, \quad (7)$$

where Φ and N are real finite functions of (r, t) . The latter condition allows us to consider the metric fields on the entire manifold $(r, t) \in \mathbb{R}^2$ without punctured points and, thus, construct geodesically complete maps of the geometries for *all* radial trajectories of test particles and light, which are continued with respect to their proper affine parameters. Recall that the solutions (7) does not diverge on the horizons $\Phi = -1/2$. It becomes obvious in the Lagrange coordinates [17]. By definition, in the $T(R)$ -regions the potential Φ is less (greater) than $-1/2$.

4 Black/white holes and cosmology

The different properties of the internal and external curvatures with respect to inversion naturally classify the models. In the T -regions, where r is the time [21], (non)invariance of (4) with respect to the change of the r sign is (non)invariance of the metrics with respect to the time inversion. The first class is that of reversible models while the second is of irreversible ones.

The first type includes eternal black/white holes. Behavior of geodesics in the space-time of the white hole is that of geodesics in the black hole space-time corrected for the time inversion (see Fig. 1). Every particle inside the black hole horizon moves in the direction of increasing curvature while in the white hole the curvature decreases on the trajectory [17]. In particular, non-linear quantum processes triggered by increasing intensity of the gravitational field are reversible in this class of models. For example, after being polarized vacuum experiences the same states again in reverse order and returns to the initial state. Identifying the gravitational field intensity with a thermodynamical potential we can talk about an analogy with reversible thermodynamical transitions.

Models of the second type deal with real irreversible processes analogous to the irreversible phase transitions (say, to the condensation of overheated vapor). Here the quantum-gravity processes generating matter in the highly variable gravitational field come into play. The matter does not disappear and remains in the T -region [22, 12]. We show (Sects. 5, 6) that in a certain class of models the singularity emerging outside the collapsing matter gives birth to a new world of the white hole which expands from high to low densities (see Fig. 2). In the course of evolution phase transitions occur and in some conditions inflation can be realized. Driven by the latter, the cosmological expansion can become isotropic within a large spatial volume. Therefore, among irreversible models there are cosmological solutions, in which a black hole (even with a small mass) gives birth to the quasi-Hubble flow of an *astrogenic* universe.

It is worth pointing out that models of both the first and second types incorporate significantly non-linear quantum processes which cannot be treated with techniques of quasi-classical perturbation theories of gravity. However, although the complete quantum theory is yet to be developed, this cannot prevent us from simulating this kind of processes phenomenologically on the basis of general physical principles (see Sects. 1, 2). Here an analogy with classical hydrodynamics jumps to mind. Many paradoxes of ideal fluid hydrodynamics were long known [23] to be solved by introducing the coefficient of viscosity even though a microscopic viscosity theory had not been developed yet.

5 A material source of the Schwarzschild metrics

In the extended Schwarzschild metrics (so-called eternal black/white hole) the singularity of black hole cannot source the gravitational field in the outer space of the black hole, since it lies in the absolute future relative to this space. Because of the non-linearity of the relativistic equations of gravity the following question is non-trivial – *does GR require, as the Newton gravity does, a central source of the curved vacuum metrics, with its properties being found from the right-hand side of the Einstein equations*²?

This problem has been discussed in the literature [24, 25, 26, 17], but an agreement is yet to be reached and the problem remains unsolved. Particularly, the authors [27] exploited a formal mathematical approach applying the technique of integrating tensor differential n -forms to evaluate the right-hand side. After some type of regularization this results in appearing of singular functionals (3D delta-functions), which, in turn, require a space to live on. In [27] this space is identified with the locally defined Minkowski space of the vierbein formalism. This choice is hard for us to agree with. First, the required flat manifold must be defined globally and, second, the singularity $r = 0$ makes impossible even the local definition because of the divergence of the vierbein.

Obviously, the issue of the extended Schwarzschild metrics source in GR arises because of the singular spacelike hypersurfaces $r = 0$ that completely reside in T -regions and are inherent to any black/white hole. For this reason we cannot be satisfied with "emergency solutions" which either remove the T -regions from consideration at all (e.g. Einstein–Rosen bridges [28], wormholes [29]) or modify them by applying *specific constraints* on gravitation or matter properties. Among the modifications are the requirements of finite maximal curvature [30, 4] or matter density [31], gravitational torsion [32] and others. I.G. Dymnikova constructed non-singular solutions with the black/white hole asymptotics at large r containing anisotropic matter, which is 'vacuum-like' in the longitudinal direction and constrained by the finite density condition (see [31] and references therein). These

²Recall that in the Newtonian physics the spherically symmetric gravitational field in vacuum is of the form $\Phi = -GM/r$, where $r = |\mathbf{x}|$. Evaluating the right-hand side of the Poisson equation in the Euclidian space \mathbb{R}^3 , one finds that the source of this field is the central mass $M = \text{const}$ located at $r = 0$: $\Delta\Phi = 4\pi GM\delta^{(3)}(\mathbf{x})$, where $\delta^{(3)}(\mathbf{x})$ – 3D delta-function.

solutions have at least *two* R -regions, an external and internal one (the latter containing the center $r = 0$), separated by a T -region. Hence, their topology dramatically differs from that of the Schwarzschild black hole. These types of solutions are beyond the scope of our consideration, since they lack the limit of the black/white hole T -region.

Let us find out which properties of the effective matter support the metric space-time. Making use of eq. (7) one obtains from the GR equations [24, 20]:

$$\Phi = -\frac{Gm}{r}, \quad (8)$$

where the finite *mass function*

$$m = m(r, t) = 4\pi \int_0^r T_t^t r^2 dr = m_0 - 4\pi r^2 \int T_t^r dt \quad (9)$$

vanishes on the inversion line $r = 0$ ($m(0, t) = 0$) thanks to the finiteness condition applied on the potential Φ . The function $m_0 = m_0(r)$ is determined by the setting of the problem. Therefore, GR informs us that the external mass in the Schwarzschild solution exists *only if there is a material source* of the metrics (1). We also conclude that for the function $m(r, t)$ to be finite, $T_t^t r^2$ must be integrable at $r = 0$. Hereafter, by definition, the distribution of pressure over r integrable in this sense is called to possess an *integrable* singularity. Mathematically, it is reminiscent of the central cusp caustics in the spatial halo distribution of density in the R -region.

Let the matter reside in a part of the T -region of the black hole³ with curvature exceeding a critical value ($T_{\mu\nu} \neq 0$ when $0 \leq r \leq r_0 < 2GM$). Assume that at low curvature the space-time is empty and described by the metrics (1) ($T_{\mu\nu} = 0$ when $r > r_0$). This situation can be thought of as a *triggered phase transition*, in which non-zero T_μ^ν components emerge at a certain value of a curvature invariant that increases near the singular hypersurface⁴.

The effective matter distribution induced by the 4-curvature of (7) outside the body of the collapsing object (hereafter, the 'star') respects the symmetry of the vacuum gravitational field (1) and, thus, possesses the global Killing t -vector. The matter is free from radial motion ($T_t^r = 0$) and its distribution is homogeneous in the space (t, θ, ϕ) and extends to the white-hole T -region ($r < 0$) lying in absolute future with respect to the maternal black hole. The generated matter in the white hole will remain in this strictly homogeneous state till it interacts with the flow of the particles coming from the surface of the collapsing 'star' (see Fig. 2). In the homogeneity region the functions $\Phi, N, p_\perp, T_I^J = \text{diag}(-p, \epsilon)$ and m depend merely on time r and we readily obtain a cosmological

³Otherwise we would obtain a star or a halo.

⁴One of the candidates to play the role of the invariant is the squared Riemann tensor $\mathcal{I} = R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}$ which in the Schwarzschild solution equals to $48(GM/r^3)^2$. In this case the time moment r_0 when the phase transition starts can be estimated from the physical dimension: $\mathcal{I} \sim 1/l_P^4$, where $l_P \approx 10^{-33}$ cm is the Planck length. For a black hole with mass of the order of that of the Sun $r_0 \sim 10^{12}l_P$.

model with the cylindrical symmetry $\mathbb{R} \times \mathbb{S}^2$ (the Kantowski-Sachs model) capable of further dynamical isotropizing to the Friedmann symmetry (see, for example, [33]). For this matter distribution the mass function

$$m(r) = -4\pi \int_0^r p r^2 dr \quad (10)$$

is odd (asymmetric) with respect to r in models of the 1st (2nd) type. The relation between p and the black or white hole mass⁵

$$M = -4\pi \int_0^{r_0} p r^2 dr \quad (11)$$

yields that if $M > 0$, at the average the longitudinal pressure is negative, $p < 0$. Note that eq. (11) does not constrain the cosmological density $\epsilon \geq 0$. The weak energy condition $\epsilon + p \geq 0$ yields the *lower* limit on the full mass (per unit t length) stored in either of the holes:

$$4\pi \int_0^{2GM} \epsilon r^2 dr \geq M. \quad (12)$$

A large universe in the white-hole T -region means both increased age and mass. One can achieve it requiring $\Phi' > 0$ ($Gm' > -\Phi > 1/2$, $-8\pi Gpr^2 > 1$), which implies the inflation condition.

The second metric potential N and the transversal pressure p_\perp of the homogeneous matter are found from the rest of the Einstein equations (the prime stands for the derivative with respect to r):

$$\frac{N'}{N} = \frac{4\pi Gr^2(\epsilon + p)}{2Gm - r}, \quad p_\perp = \frac{N'}{2N} \left(\frac{m}{4\pi r^2} - r\epsilon \right) - \frac{(r^2\epsilon)'}{2r}, \quad (13)$$

where the function $m = m(r)$ satisfies eq. (10). Excluding N'/N in (13) we obtain the energy conservation law. In order to integrate this equation it is necessary to specify the effective matter Lagrangian or its equation-of-state.

6 Models with integrable singularity $r = 0$

To give an example we consider 1st type models of eternal black/white holes with triggered material sources with the Lagrangian density that depends on the intrinsic curvature $\rho = r^{-2}$ of the 2-surfaces $(r, t) = \text{const}$, $\mathcal{L}_m = p(\rho)$. Variation of the action with respect to the metrics (4, 5) yields⁶ $\epsilon = -p$, which means that in the matter the function N also remains constant enabling us to choose $N = 1$ (see (13)).

⁵We assume that the matter in the white hole is in the region $0 < -r \leq r_0 \leq 2GM$, where $M = M_{WH} = -4\pi \int_0^{r_0} p(-r) r^2 dr$. In the 1st (2nd) type models the black and white hole masses M as well as the r_0 parameters are equal (different).

⁶Evidently the following relation is not universally true. For example, the functional dependance $\mathcal{L}_m = p(\Phi, \rho)$ yields $\epsilon + p \neq 0$. For more general models see [20].

Let us consider the power-law profiles of the matter density

$$\epsilon = p_0 e \cdot \theta(1 - x^2), \quad e = e(x, \alpha) \equiv \frac{|x|^{-2\alpha} - 1}{\alpha}, \quad (14)$$

which are continuous on the borders $x = \pm 1$, where the variable $x \equiv r/r_0$, the function $\theta(y) = \int_{-\infty}^y \delta(y) dy$, the exponent $\alpha = \text{const} < 3/2$ and the parameter p_0 is related to the external mass as follows:

$$M = 4\pi r_0^3 \int_0^1 \epsilon(x) x^2 dx = \frac{8\pi p_0 r_0^3}{3(3 - 2\alpha)}.$$

The relativistic equations yield the transversal pressure

$$p_{\perp}(x) = p_0 [1 + (\alpha - 1)e] \cdot \theta(1 - x^2), \quad (15)$$

and the metric potential (8)

$$\Phi(x) = \begin{cases} -\frac{GM}{r_0} x^2 \left(1 + \frac{3}{2}e\right), & |x| < 1 \\ -\frac{GM}{|r|}, & |x| \geq 1 \end{cases}. \quad (16)$$

These distributions satisfy the weak energy condition while the dominant one is broken on the matter border where the energy density tends to zero while the transversal pressure is finite. Note also the delta-like feature of the distributions⁷ (14)-(16) as $r_0 \rightarrow 0$ ($\bar{\delta}(r) \equiv (4\pi r^2)^{-1} \delta(r)$):

$$\epsilon(x) \xrightarrow{r_0 \rightarrow 0} 2M\bar{\delta}(r), \quad p_{\perp}(x) \xrightarrow{r_0 \rightarrow 0} M\bar{\delta}(r), \quad (17)$$

which proves the source of the Schwarzschild eternal black/white hole to be localized on the hypersurface $r = 0$. As mentioned above, this matter *alone* cannot source the black hole, the latter residing in the absolute past with respect to the singular hypersurface. It is sourced by a similar matter located on the other singular hypersurface, which lies under the point of intersection of the horizons $r = 2GM$ (see Fig. 1). In other words, the Penrose diagram of the eternal black/white hole is an infinite chain of elementary Penrose diagrams engaged by material regions that source the geometry.

With $\alpha = 1$, as $r \rightarrow 0$ the potential tends to a constant, $-3GM/2r_0$, and we obtain an integrable singularity with the divergent density $\propto r^{-2}$ and the 'hat' transversal pressure profile, $p_{\perp} = p_0 = \text{const}$ at $|r| < r_0$. Following [25] to study the story of an extended body falling to $r = 0$, let us show that unlike the Schwarzschild solution tidal forces in the

⁷Evaluating the limit we retain the mass to be constant:

$$\int_{-r_0}^{r_0} \epsilon(x) r^2 dr = 2 \int_{-r_0}^{r_0} p_{\perp}(x) r^2 dr = \frac{M}{2\pi}.$$

The ratio $\epsilon/p_{\perp} \xrightarrow{r_0 \rightarrow 0} 2$ is determined by the equation-of-state and, in general, arbitrary.

model with $\alpha = 1$ remain constant as the body proceeds towards the singular hypersurface which prevents mechanical disruption. Indeed, non-vanishing components of the Riemann tensor in a locally inertial reference frame $(\hat{t}, \hat{r}, \hat{\theta}, \hat{\varphi})$ are given by the formulae:

$$R_{\hat{t}\hat{r}\hat{t}\hat{r}} = \Phi'', \quad R_{\hat{t}\hat{\theta}\hat{t}\hat{\theta}} = R_{\hat{t}\hat{\varphi}\hat{t}\hat{\varphi}} = \frac{\Phi'}{r}, \quad (18)$$

$$R_{\hat{\theta}\hat{\varphi}\hat{\theta}\hat{\varphi}} = -\frac{2\Phi}{r^2}, \quad R_{\hat{r}\hat{\theta}\hat{r}\hat{\theta}} = R_{\hat{r}\hat{\varphi}\hat{r}\hat{\varphi}} = -\frac{\Phi'}{r}. \quad (19)$$

$R_{\hat{\theta}\hat{\varphi}\hat{\theta}\hat{\varphi}}$ is the unique component which diverges on the solution (16) with $\alpha = 1$ as r tends to zero. It, however, does not enter the equation which governs how fast two freely moving particles separated by the spatial vector $\xi^{\hat{a}}$ and staying at rest in the locally inertial reference frame accelerate one relative to the other, $\hat{a} = (\hat{t}, \hat{\theta}, \hat{\varphi})$:

$$\frac{D^2 \xi^{\hat{a}}}{d\hat{r}^2} = -R_{\hat{r}\hat{a}\hat{r}\hat{b}} \xi^{\hat{b}} \sim \frac{GM}{r_0^3} \xi^{\hat{a}}.$$

Therefore, the singularity $r = 0$ can be passed through and allows us to extend the geodesics into the white-hole region.

7 Discussion

To summarize, we have shown that there exist geodesically complete eternal black/white holes geometries with regular X -space (see eqs. (7)-(10), Fig. 1) and constructed a toy model of the geometry which depends on the intrinsic curvature of the Y -space (see Sect. 6).

Based on this we argue that in the introduced class of spherically symmetric GR solutions with 2+2 split and integrable singularity $r = 0$ in black/white holes there are solutions containing cosmological subsystems in the T -regions. These include expanding homogeneous matter flows with the spatial symmetry $\mathbb{R} \times \mathbb{S}^2$. Driven by inflation they can grow to become a Hubble flow whose parameters may be similar to those of our Universe. In case the initial black hole originates from collapse of a compact astrophysical object we call such subsystem astrogenic universe. A toy model of this kind is given in [20].

Particularly, we would like to discuss the possibility of modelling such processes. The Einstein equations imply all physical degrees of freedom to be separated into spacetime and material ones. In fact, effects of quantum gravity 'mix' the degrees of freedom and the dynamical equation contain merely the expectations $g_{\mu\nu}$ and $T_{\mu\nu}$ averaged over the states of perturbation fields which are vacuum states in the static R -region. As well known, in the quasiclassical limit the effects of quantum gravity as well as their backreaction are negligible. However, at large curvature it is true no more. The emergent (non-vanishing) effective energy-momentum tensor essentially rebuilds the original metrics extending it and generating the 'new' space-time residing in the absolute future relative to the 'old'

world. Since the science does not have general equations at its dispose yet while quantum corrections following from perturbation theories contain little information about the full picture, we simulate the 'mixing' in GR phenomenologically. As exemplified in spherically symmetric models we smooth out spacetime distributions (the finiteness condition for $g_{\mu\nu}$) and consider profiles of the effective tension $T_{\mu\nu}$ with moderate (integrable) divergence at large curvature ($r \rightarrow 0$).

Note that the maternal matter of the collapsing stars themselves does not constitute the astrogenic universes. It rather triggers their generation, which dramatically differs our paradigm of cosmic flow generation from baby-universe-like solutions and bouncing models. Indeed, the collapsing matter is necessary to induce the large curvature, which, in turn, triggers particle generation in the vicinity of the singular hypersurface (see Fig. 2). If we assume that the symmetry of the Schwarzschild solution holds inside the generated matter, the global similar symmetry with Killing t -vector will be respected in the white hole as well. Moreover, clear is the origin of the impulse that launches the expansion – it is the momentum accumulated in the course of the collapse which is transferred with time outside the collapsing object to the expanding white-hole effective matter thanks to long-range tidal nature of gravitation.

Emphasis to be made is on extension of geodesics. The point is the question whether it is possible to extend geodesics in a solution has nothing to do with the existence of an astrogenic universe (see the previous paragraph). The affirmative reply would only mean that signals from the mother Universe can pass to the daughter world.

The suggested hypothesis is attractive from the physical point of view, since it allows one to relate two, perhaps, most famous GR solutions, collapse and anticollapse (the cosmological expansion) and resolve the initial value problem in cosmology (including the initial symmetry) with the aid of the energy-momentum conservation law. This paradigm of cosmogenesis requires, however, further research – both new models and observational tests are to be found.

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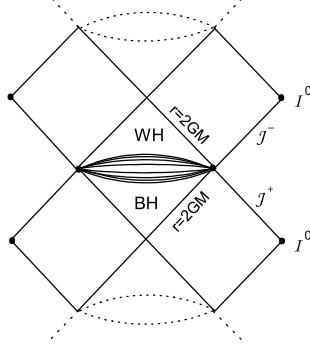


Figure 1: Penrose diagram of eternal black/white hole. The matter (shaded region) separates contracting (BH) and expanding (WH) T -regions

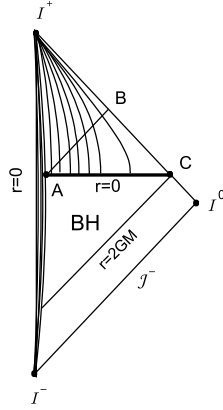


Figure 2: A sketch of the diagram of an astrogenic universe ($M_{WH} \gg M$). ABC is the region of spatially homogeneous cosmology